Basic Electromagnetism

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Contents

Preface

Electricity and magnetism are basic to our understanding of the properties of matter and yet are often regarded as the difficult parts of an undergraduate course in physics, materials science or engineering. In the first six chapters of this book answers are developed from first principles to such questions as: What is electricity? What is electromagnetism? Why are some materials magnetic and others non-magnetic? What *is* magnetism?

These questions can be answered in two related ways. On the one hand the *classical* explanation is in terms of classical concepts: electric charge (q), electric and magnetic fields ($\boldsymbol{E}$ and $\boldsymbol{B}$) and electric currents. On the other hand the *microscopic* (or 'atomic') explanation is in terms of quantum concepts: electrons, nuclei, electron orbits in atoms, electron spin and photons. Microscopic explanations underlie classical ones, but they do not deny them. The great triumphs of classical theory are mechanics, gravitation, thermodynamics, electromagnetism and relativity.

Historically the classical theories began at the time of Newton (seventeenth century) and were completed by Maxwell (nineteenth century) and by Einstein (early twentieth century). Microscopic explanations began with J.J. Thomson's discovery of the electron in 1897. For most physical phenomena it is best to seek a classical explanation first, especially for phenomena at room temperature, or low energy, when quantum effects are small. This book presents classical theory in a logical, self-consistent sequence, but reference is made to microscopic (quantum) theory at each appropriate stage.

Electromagnetism began in 1819 with the discovery by Oersted that an electric current is associated with a magnetic field and was followed in 1820 by Ampère's discovery that two wires carrying electric currents exerted magnetic forces on one another. But it was Faraday's discovery of electromagnetic induction in 1831, or as he put it, the conversion of magnetism into electricity, that finally showed that electricity and magnetism were not distinct, separate phenomena, but interacted when there were time-varying electric or magnetic fields. The beauty of electromagnetism is that Faraday's experiments led to a symmary of the whole of electromagnetism in just the four equations of Maxwell's theory, which relate $\boldsymbol{E}$ and $\boldsymbol{B}$ in space with fixed and moving charges, together with the electromagnetic force law. These equations are greatly simplified when we deal with *statics*, that is, variables that do not depend on time t, or *stationary* variables. Maxwell's equations then simplify and separate into two independent pairs of equations:

1. The first pair describe the electrostatic field $\boldsymbol{E}$ for fixed charges and are known as *Gauss's law* and the *circulation law.* They summarize electrostatics.
2. The second pair describe the magnetostatic field $\boldsymbol{B}$ for steady currents (charges moving at constant speed) and are known as *Gauss's law* and *Ampère's law.* They summarize magnetostatics.

In electrostatics only the $\boldsymbol{E}$ field appears and $\partial E/\partial t = 0$; in magnetostatics only the $\boldsymbol{B}$ field appears and $\partial B/\partial t = 0$. So under these conditions, electricity and magnetism *are* classically distinct, separate phenomena. But if you charge a capacitor (q varying with time) or move a magnet ($\boldsymbol{B}$ at a point varying in time) then $\boldsymbol{E}$ and $\boldsymbol{B}$ are no longer independent and new terms in the equations due to *electromagnetism* appear, as first discovered by Faraday (*Faraday's law*) and Maxwell (*Maxwell's law*).

The development of the subject in this text is therefore first electrostatics, then magnetostatics, followed by electromagnetism and magnetism. The seventh chapter summarizes electromagnetism in terms of Maxwell's equations, which are then used to study the propagation and generation of electromagnetic waves. The first seven chapters often comprise a first course in electromagnetism for undergraduates.

In studying the solutions of Maxwell's equations you will find answers to such questions as: What is an electromagnetic wave? Why does a radio wave travel through space at the speed of light? How is a radio wave generated? Why does light pass through a straight tunnel when a radio wave does not? How does light travel down a curved glass fibre?

Before studying these solutions, Chapter 8 discusses the remarkable fact that the classical laws of electromagnetism are fully consistent with Einstein's special theory of relativity. The following four chapters provide solutions of Maxwell's equations for the propagation of electromagnetic waves in free space, in dielectrics, across interfaces and in conductors, respectively. In Chapter 13 the generation of radio waves from dipoles and of microwaves from other antennas is explained, while the final chapter shows how these waves can be transmitted down waveguides and coaxial lines. In conclusion, the use of resonant and re-entrant cavities leads to a discussion of the classical theory of radiation and its usefulness as a limiting case of the quantum theory of radiation.

The spectrum of electromagnetic waves covers an enormous range of frequencies, from the very low frequencies (VLF), used to communicate with submerged submarines, to the enormous frequencies (10^{24} hertz) associated with some cosmic rays from outer space. The complete spectrum is illustrated in Appendix D, where it is characterized by both the classical, wave properties of frequency (ν) and wavelength (λ) and the quantized, photon properties of energy ($h\nu$) and temperature ($h\nu/k_B$). Classical electromagnetism provides a theory of the wave properties of radiation over a wide frequency range, including, for example, the diffraction of X-rays by crystals, but for the interactions of radiation with matter classical theory only applies in the long wavelength, low frequency, low energy ($h\nu \ll k_B T$) limit. The generation of electromagnetic radiation is similarly the classical process of acceleration of electrons in producing a radio wave, where the wavelength is macroscopic, but quantum processes are involved in the production of X-rays by electronic transitions in atoms, or gamma rays by nucleonic transitions in nuclei, where the wavelength are microscopic. The production of light by laser action is an interesting example of the combination of the classical process of reflection with the quantum process of stimulated emission. In this book the limits of classical electromagnetism are explained and the usefulness of the wave and particle properties of radiation are discussed, so that the reader is provided with an understanding of the applicability and limitations of classical theory.

The international system of units (SI units) are used throughout and are listed for each electromagnetic quantity in Appendix A. Since Gaussian units are still used in some research papers on electromagnetism, Appendix B lists Maxwell's equations in these units and states the conversion from the Gaussian to the SI system. The physical constants used in the text are listed in Appendix C with their approximate values and units. In Appendices E and F there are summaries of the most useful relations in vector calculus and special relativity. Finally, each chapter, except Chapters 1 and 7, has a set of associated exercises in Appendix G with answers in Appendix H.

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This book owes much to the many undergraduates who have participated in my tutorials and lectures on electromagnetism at the Universities of Cambridge, Lancaster and London for more than twenty years. Of the numerous texts I have consulted, none has been so illuminating as Volume 2 of the *Feynman Lectures on Physics* by R.P. Feynman, R.B. Leighton and M. Sands (Addison-Wesley, London, 1964).

This new, revised edition is a combination of my *Electricity and Magnetism* (Routledge & Kegan Paul, London, 1984) and my *Electromagnetic Waves* (Routledge & Kegan Paul, London, 1985). It is a pleasure to thank colleagues in the Universities of London and Sussex for their helpful comments, especially Professor E.J. Burge for a number of improvements to *Electricity and Magnetism*, after he had used it for a first course to undergraduates in Physics at Royal Holloway, University of London. Thanks are due to the University of London for permission to reproduce some problems (marked *L*) from BSc course unit examinations taken at the end of their first year by students reading Physics at Bedford College.

It is a plesure to thank Mrs Sheila Pearson for her accurate and rapid typing of the original manuscripts and my wife for her constant support and encouragement over many years.

List of symbols

$\boldsymbol{A}$	Magnetic vector potential
A	Area
$\boldsymbol{B}$	Magnetic field
B	Susceptance
C	Closed loop; capacitance
c	Speed of light
$\boldsymbol{D}$	Electric displacement
d	Distance
$\mathscr{E}$	Electromotive force
$\boldsymbol{E}$	Electric field
e	Electronic charge
$\mathscr{F}_m$	Magnetomotive force
$\boldsymbol{F}$	Force
$\boldsymbol{F}_E$	Electrical force
$\boldsymbol{F}_G$	Gravitational force
f	Focal length
G	Gravitational constant; conductance
$\boldsymbol{H}$	Magnetizing field
$\boldsymbol{H}_m$	Demagnetizing field
h	Planck constant
I	Electric current
I_m	Surface magnetizing current
$\hat{\mathbf{i}}$	Cartesian unit vector
$\boldsymbol{i}$	Surface current density
i_f	Solenoidal surface current density
$\underset{\sim}{i}_m$	Magnetization surface current density
$\hat{\mathbf{j}}$	Cartesian unit vector
$\boldsymbol{j}$	Electric current density

$\boldsymbol{j}_{\mathrm{f}}$	Conduction current density
$\boldsymbol{j}_{\mathrm{m}}$	Magnetization current density
$\boldsymbol{j}_{\mathrm{p}}$	Polarization current density
$\hat{\mathbf{k}}$	Cartesian unit vector
$\boldsymbol{k}$	Wave vector
k	Coupling coefficient; wave number
k_{g}	Waveguide wave number
k_{B}	Boltzmann constant
$k_{\mathrm{R}}, k_{\mathrm{I}}$	Real and imaginary parts of $k = k_{\mathrm{R}} - \mathrm{i}k_{\mathrm{I}}$
L	Self inductance
l	Length
$\mathbf{d}\boldsymbol{l}$	Electric current element
$\boldsymbol{M}$	Magnetization
M	mutual inductance
$\boldsymbol{m}$	Magnetic dipole moment
m	Mass
N	Number density; total number of turns
$\boldsymbol{n}$	Unit normal vector
$\hat{n}$	Number of turns per unit length; refractive index
$n_{\mathrm{R}}, n_{\mathrm{I}}$	Real and imaginary parts of $n = n_{\mathrm{R}} - in_{\mathrm{I}}$
$\boldsymbol{P}$	Electric polarization
P	Total radiated power
P_{n}	Legendre function
$\boldsymbol{p}$	Electric dipole moment; electron momentum
$\boldsymbol{p}_0$	Permanent electric dipole moment
p_{r}	Radiation pressure
Q	Total electric charge; quadrupole moment; quality factor
q	Electric charge
$\mathscr{R}$	Reluctance
R	Electrical resistance; reflectance
R_0	Reflectance at normal incidence
R_{r}	Radiation resistance
$\boldsymbol{r}$	Distance vector
r	Distance; cylindrical or spherical radius
r_0	Classical electron radius
$\mathscr{S}$	Poynting vector, electromagnetic energy flux
$\boldsymbol{S}$	Surface area vector
$\boldsymbol{s}$	Distance vector
s	Distance; contour
$\boldsymbol{T}$	Torque vector
T	Temperature; transmittance

T_C	Curie temperature
T_F	Fermi temperature
t	Time
U	Energy
u	Energy density; relative velocity
V	Volume; potential difference
$\boldsymbol{v}$	Velocity vector
v_F	Fermi velocity
W	Mechanical work
x	Cartesian coordinate
Y	Admittance
y	Cartesian coordinate
Z	Wave impedance; atomic number
Z_0	Wave impedance of free space
z	Cartesian coordinate
α	Polarizability
β	Absorption coefficient
γ	Electrical conductivity; damping constant
δ	Loss angle; skin depth
ε	Absolute permittivity
ε_0	Electric constant; permittivity of free space
ε_r	Relative permittivity; dielectric constant
$\varepsilon_R, \varepsilon_I$	Real and imaginary parts of $\varepsilon = \varepsilon_R - i\varepsilon_I$
ε_s	Static value of relative permittivity
ε_∞	High frequency limit of relative permittivity
θ	Spherical polar coordinate
θ_c	Critical angle (of incidence)
λ	Electric charge per unit length; wavelength
λ_c	Cut-off wavelength of waveguide
λ_g	Waveguide wavelength of waveguide
λ_0	Free space wavelength of radiation
$\boldsymbol{\mu}$	Magnetic moment of a particle
μ_B	Bohr magneton
μ	Absolute permeability
μ_0	Magnetic constant; permeability of free space
μ_r	Relative permeability
ν	Frequency
ρ	Electric charge density
ρ_f	Free charge density
ρ_p	Polarization charge density
σ	Surface charge density

σ_f	Surface density of free charges
σ_p	Surface density of polarization charges
σ_0	Resonant scattering cross-section
σ_R	Rayleigh scattering cross-section
σ_T	Thomson scattering cross-section
τ	Volume of integration; mean time between collisions
Φ	Magnetic flux
ϕ	Electric (scalar) potential; cylindrical polar coordinate; phase angle of a wave
χ_e	Electric susceptibility
χ_m	Magnetic susceptibility
ψ	Spherical polar coordinate; azimuthal angle; wave function
Ω	General scalar function
$d\Omega$	Element of solid angle
ω	Angular velocity of frequency
ω_c	Cut-off (angular) frequency of waveguide
ω_L	Larmor (angular) frequency
ω_0	Resonant (angular) frequency
ω_p	Plasma (angular) frequency

1

Introduction

Although all electromagnetic phenomena can be studied in empty space, an important part of any introductory course on electricity and magnetism is a proper understanding of the nature of matter. We shall therefore discuss dielectric behaviour in the chapter on electrostatics, conduction in metal wires in that on magnetostatics, and magnetism in matter (whether para-, dia- or ferro-magnetism) in the chapter on magnetism. In this first chapter the nature of matter is summarized.

All matter is composed of elementary particles, some charged positively (protons), some charged negatively (electrons) and some without charge (neutrons). The forces between these particles are of three different sorts – gravitational, electrical and nuclear – which differ enormously in their strength and range.

The *gravitational force* was made famous by Newton in his studies of the planets and expressed by him in 1686 in his law of universal gravitation that 'every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them'. This is the first inverse square law of force which, for two masses m_1 and m_2, is given by

$$F_G = \frac{Gm_1m_2}{r^2} \tag{1.1}$$

where r is the distance between m_1 and m_2 and G is the gravitational constant. The *electrical force* will be familiar as the law Coulomb found in 1785 for the force between electrical charges. This is another inverse square law of force. If r is now the distance between the charges q_1 and q_2, and K is an electrical constant then

$$F_E = \frac{Kq_1q_2}{r^2} \tag{1.2}$$

The third type of force between the elementary particles that constitute matter is a comparatively recent discovery. In 1932 Chadwick found that the nuclei of atoms and molecules contained not only protons but new particles – neutrons – and so there had to be a third type of force, the *nuclear force*, that held these particles together in the nucleus. Collectively the particles in the nucleus are known as nucleons.

This nuclear force, composed of both weak and strong interactions, is exceedingly short range. For example the nuclear force decreases in some cases as $r^{-2}\exp(-r/r_0)$, where r_0 is about $1\,\text{fm} = 10^{-15}\,\text{m}$. An introduction to the exciting properties of nuclei and elementary particles can be found in the book by Professor Blin-Stoyle in this series. In contrast the gravitational and electrical forces are comparatively long range (Fig. 1.1). It is obvious from the motion of the planets round the sun that gravitational forces are long range. It is not so obvious that electrical forces are similarly long range because electrical charges are usually screened by other charges of opposite sign at comparatively short range, so that the overall effect at long range is negligible.

Although both the gravitational and electrical force obey an inverse square law, their strengths differ enormously. For the proton–electron pair which comprises the hydrogen atom, the electrical force F_E is about 10^{39} or one thousand million million million million million million times as strong as the gravitational force F_G, as can

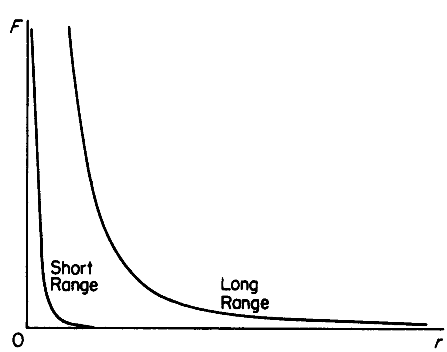

Fig. 1.1 Short-range and long-range forces.

be easily shown from (1.1) and (1.2) and a knowledge of the constants. So we can nearly always neglect gravitational effects in the presence of electrical forces. The exception is experiments like Millikan's oil drop, where the enormous mass of the earth acts on the oil drop with a force comparable to the electrical one exerted on the tiny charges of the oil drop as it moves between the charged plates.

The gravitational and electrical forces also differ in one other important respect: the gravitational force between particles of ordinary matter is always attractive, whereas the electrical force is repulsive (positive) between like charges and attractive (negative) between unlike charges. The net result is that large masses have large gravitational attractions for one another, but normally have negligible electrical forces between them.

The paradox is that although all matter is held together by electrical forces, of the interatomic or intermolecular or chemical-bond types, which are immensely strong forces, large objects are electrically neutral to a very high degree. The electrical balance between the number of protons and electrons is extraordinarily precise in all ordinary objects. To see how exact this balance is, Feynman has calculated that the repulsive force between two people standing at arm's length from each other who each had 1% more electrons than protons in their bodies would be enormous – enough, in fact, to repel a weight equal to that of the entire earth! So matter is electrically neutral because it has a perfect charge balance and this gives solids great stiffness and strength.

The study of electrical forces, electromagnetism, begins with Coulomb's law, (1.2). All matter is held together by the electromagnetic interactions between atoms, between molecules and between cells, although the forces holding molecules and cells together are more complicated than the simple Coulomb interaction. The studies of condensed state physics, of chemistry and of biology are thus all dependent on an understanding of electromagnetism. This text develops the subject from Coulomb's law to Maxwell's equations, which summarize all the properties of the electromagnetic fields, in free space and matter. But if you ask why does the strong electrical attraction between a proton and an electron result in such comparatively large atoms rather than form a small electron–proton pair, you will not find the answer in Maxwell's equations alone. The study of electrical forces between particles at atomic or subatomic distances requires a new physics, quantum mechanics, which is the subject of the book by Professor Davies in this series.

2

Electrostatics

Electric charge has been known since the Greeks first rubbed amber and noticed that it then attracted small objects. Little further progress was made until the eighteenth century when du Fay showed that there were two sorts of charge. One sort followed the rubbing of an amber rod with wool, the other a glass rod with silk. It was Benjamin Franklin who arbitrarily named the latter a positive charge and the original amber one a negative charge. He also showed that the total charge in a rubbing experiment was constant.

2.1 COULOMB'S LAW

In 1785 Coulomb succeeded in discovering the fundamental law of electrostatics. A brilliant experimenter, he was able to invent and build a highly sensitive torsion balance with which he could measure precisely the relative force of repulsion between two light, insulating, pith balls when charged similarly and placed at different distances apart. He showed that this electrostatic force:

1. acts along the line joining the particles;
2. is proportional to the magnitude of each charge; and
3. decreases inversely as the square of the distance apart.

It is therefore a long-range force (Fig. 1.1) and is given by the vector equation for the force $\boldsymbol{F}_1$ on charge q_1 due to charge q_2. Thus,

$$\boldsymbol{F}_1 = \frac{Kq_1q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} \tag{2.1}$$

where $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $r_{12}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$ and $\hat{\mathbf{r}}_{12}$ is a unit vector drawn *to* 1 *from* 2 (Fig. 2.1) given by $\boldsymbol{r}_{12}/r_{12}$. Fig. 2.1

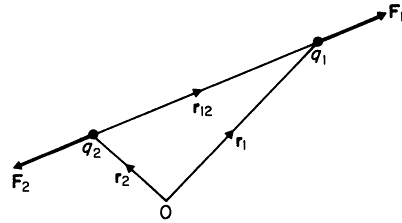

Fig. 2.1 Electrostatic forces between electric charges.

shows that Newton's laws must apply and the force $\boldsymbol{F}_2$ on charge q_2 due to q_1 is $\boldsymbol{F}_2 = -\boldsymbol{F}_1$. When both charges have the same sign, the force acts positively, that is the charges are repelled, while between a negative and a positive charge the force acts negatively and the charges are attracted.

For historical reasons the constant of proportionality K in (2.1) is not one, but is defined as

$$K = \frac{1}{4\pi\varepsilon_0} = 10^{-7}c^2 \qquad (2.2)$$

where ε_0 is the *electric constant* (permittivity of free space) and c is the velocity of light. The constant has to be determined from experiment. A recent value of $c = 2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}}$ is accurate to better than 1 in 10^6, but for use in problems can be taken as $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. On the same basis $K = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$, using the SI unit coulomb (C) for electric charge.

It is important to note that we have written in (2.1) Coulomb's law for charges in a *vacuum*; we have not mentioned the effects of a dielectric or other medium.

2.1.1 Principle of superposition

The only other basic law in electrostatics is the principle of superposition of electric forces. The principle states that if more than one force acts on a charge, then all the forces on that charge can be added vectorially into a single force. Thus for the total force on a charge q_1 due to charges q_2 at $\boldsymbol{r}_{12}$, q_3 at $\boldsymbol{r}_{13}$, etc., we have:

$$\boldsymbol{F}_1 = \boldsymbol{F}_{12} + \boldsymbol{F}_{13} + \cdots$$